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QUANTITIES

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The world contains more than just individuals, together with properties and relations holding amongst them.

Individuals either exist, or they don’t: they do not come in degrees. Likewise, properties and relations are either instantiated by given things, or they aren’t: they cannot be partly instantiated or barely instantiated, or almost fully instantiated, and so on.

Yet some things in the world seem not to be quite like that. Individuals can be counted: but a substance, like plutonium for instance, cannot be counted, but only measured. We can say “how much”, but not “how many”. Individuals are reflected in language by so-called count nouns like “door” and “key”; substances correspond to mass terms like “bread” and “water”.

Properties and relations, too, divide into two sorts. With some we need only say whether given things do or do not possess the properties or relations in question. But there are others which seem to come in amounts, and with these the simple “on” or “off” of being instantiated or not being instantiated seems to leave something out. There is more to be said, with such properties and relations, than whether they are instantiated by a particular individual or individuals or not. Consider pleasure. Even after specifying what kind of pleasure is on offer, you may want to know more. You may want to know how much. Consider also mass, charge, or any other such basic physical property. To specify the mass of something, it is not enough to say whether the thing does, or does not, have mass: you must also say how much mass it has. Similarly with a relation like relative velocity, it is not enough to say whether one object has velocity relative to another object: we also want to know how quickly the relative positions are changing. We use the term quantities for properties like mass, and relations like relative velocity, which seem to come in amounts.

We will be concentrating on quantities in this sense, and not on the problem of mass terms and substances. There are, however, analogies between the problem of substances and the problem of quantities (in our sense); and it may be worth keeping some of these in mind. There is in existence a more or less orthodox account of mass terms and substances; there is no equally well-entrenched account of quantities. Our account of quantities will, however, be an “orthodox” one at least in the sense that it is a natural analogue of the orthodox account of mass terms and substances.

In explaining the nature of quantities, we are drawn to the core of traditional metaphysics. The theory of universals (that is, of properties and relations) arises from a recognition that, in some sense, two distinct things may be both “the same”, and “different”, at the same time. This is superficially a contradiction.

The conflict evaporates, however, when we say that the things which are the same in one respect may yet be different in other respects. Different properties or relations constitute those different “respects”.

Quantities cause problems, because it seems as though two things may be both “the same” and “different” — in the very same respect. Two things may be the same, in that both have mass; yet they may be different, in that one has more mass than the other.

Not only are quantities possessed to a greater or lesser extent: it is also possible to specify how much greater. We may say that the mass of A is closer to that of B than it is to that of C. And of course in some cases, we go on to measure this difference. A may be twice as massive as B, and with the specification of a scale with a standard unit of mass, we may say that A is 2.4 kg and B is 4.8 kg.

PLATONIC THEORIES

Plato, it seems, was aware of the problem of coping with quantities, within the context of a theory of universals (see especially the Philebus). If a property is something an individual either has, or does not have, with no intermediate possibility — then it is hard to handle quantities.

Plato's own theory, in contrast, leaves much more space for quantities. There are things called “Forms”, which exist independently of
individuals; and individuals do not simply have, or not have a Form. Rather, an individual may resemble, or “participate in” a Form to different degrees. Or, to put it slightly differently: an individual can approach an Ideal more or less closely.

We have no intention of being drawn into scholarly reflections on the interpretations of Plato. Yet we will nevertheless use his name as a label for a class of related theories, reminiscent of Plato. All these theories account for quantities by positing one entity; and a varying relationship between this entity and individuals. The former entity explains what various individuals have in common; and the latter, varying relationship explains the degrees to which quantities are manifested by individuals.

When abstractly set out, this class of theories subsumes a form of nominalism, championed by Berkeley. In this form of nominalism, a platonic, abstract Form is replaced by a selected individual called a paradigm. What a variety of individuals have in common, is that they all resemble the same paradigm. The idea is that an individual either does, or does not, bear sufficient resemblance to a paradigm. Properties give way to paradigms, together with a threshold of “sufficient resemblance”. It is less often noted, however, that there is no need to restrict our attention to such a threshold. We may take note of degrees of resemblance to a paradigm. And these may play the role of the degrees of “participation”, in the platonic theory of Forms.

All theories of this platonic, or berkeleyan, kind share a noteworthy feature. They explain how a property can come in degrees, by appeal to a relationship that comes in degrees. It does not matter whether the relationship is called “participation”, in memory of Plato; or “resemblance”, in memory of Berkeley. The point remains, that it is a relationship that comes in degrees.

This means that the central problem, posed by quantities, stands as yet unresolved. The problem is, to explain how things can be the same, and yet different, in the same respect. In platonic theories, we are faced with various pairs which are all the same (in that they are all instances of resemblance to a particular paradigm, or participation in a particular Form) and yet at the same time different (in that one pair displays more resemblance, or participation, than another). Other quantities have been explained away, in terms of a single unexplained quantity. And the
notion of 'degrees of a relationship' cries out for analysis, and the platonist offers none.

So as yet, no explanation has been given, of just what a quantity is, or how we can resolve the apparent contradiction in things being "the same, yet different, in the same respect".

We do not claim that platonic theories are mistaken: only that they are incomplete. There may indeed be some single entity, to which individuals are more, or less, closely related. We claim only that, by itself, such a theory fails to explain what we are setting out to explain — the nature of quantities.

DETERMINABLES AND DETERMINATES

The platonic strategy posits a single entity, together with varying relationships of individuals to that entity.

An alternative strategy posits many properties — one for each specific degree of a quantity. Thus for instance, it is posited that the predicates "having a mass of 2.0 kg", "having a mass of 4 kg", and so on, all correspond to distinct non-overlapping properties.

Plato's strategy makes it easy to say what various objects, with distinct masses, have in common: it is less easy to say what makes them different. The alternative strategy easily handles what Plato finds difficult. It easily explains what makes objects with distinct masses different: they differ because each has a property the other lacks. Yet Plato's strategy does explain something which the alternative strategy seems not to handle as well. Things with different masses do have something in common. To adequately reflect this, the alternative theory must add another property to the picture — one which two objects with distinct masses have in common. This property, which they share, is called a determinable; the two distinct properties, not shared by the two objects, are called determinates. The determinable is, in this example, "(having) mass"; the determinates are "(having) this much mass" and "(having) that much mass", or "(having) mass of so-and-so kg" and "(having) mass of such-and-such kg".

Mass provides, of course, just one example of determinables and determinates. Colour is also a determinable; and red, green and so on are determinates. Note also that, although red is a determinate, relative
to the determinable of colour, nevertheless, red is also a determinable, relative to the more specific determinate shades of red. And pains, too, display the same spectrum of problems. (Plato's *Philebus* focusses on pleasures; but modern discussions tend to concentrate more on pains.) Pain is a determinable; throbbing pain is, relative to that determinable, a determinate. Pain in dolphins will also count as a determinate relative to that determinable, as will pain in humans. Furthermore, within the category of, say, burning pain in humans, there will be determinate degrees or intensities.

Much attention has been focussed on pains, as a representative example of a mental state, especially in recent discussions of functionalism. Colours, too, have been much under the spotlight, as representative examples of secondary qualities. The fundamental quantities in physics, the primary qualities, have been less extensively discussed. In examining the problem of quantities, we can draw on ideas from these other areas; and we can test a theory of quantities by seeing how smoothly it operates in each of these applications. We shall not pursue the details here; but it is worth noting that the treatment of determinables and determinates given so far generally proves too restrictive in such applications. Variants are called for, of the sorts we will be discussing in the following sections.

For now we note that positing both determinates and determinables gives answers both to what is different and what is in common. Objects having the one determinable can have different determinates. And yet this account still seems unsatisfactory. Either the relationship between determinable and determinates is objective or it is not. If there is some objective relationship between the determinate and determinable, which seems plausible, we need some explication of what it is. Yet none is offered. While if there is no objective connection then it becomes stipulative to claim that the objects falling under various determinates will share some "independent" determinable. Again there is an incompleteness in this approach which a satisfactory theory must complete.

**SECOND-ORDER VARIANT**

The above account of determinables and determinates is modelled on the position set out by W. E. Johnson.³ In Johnson's version, the
determinable and the determinate are both properties of individuals. Thus, for instance, the object which is coloured (a determinable) is the very same object which is, say, blue (a determinate).

It is possible, however, to approach the matter somewhat differently. We begin with a distinct property for each individual with a different determinate. That is, for instance, we begin with a battery of different properties, one for each precise shade of colour. Then we note that all these properties have something in common. Each distinct shade is a shade of colour. There is a property, that of “being a colour”, which each determinate shade shares with each other determinate shade. This common property is a property of properties — a second-degree property.

The move we suggest here is analogous to the orthodox treatment of substances, in the following way. Two puddles of water are the same, and yet different: they are different individuals, but they have a common property. Similarly, determinate properties are the same, and yet different: they are different properties, but they have a common property.

How does this theory fare over the central problem for quantities — the problem of explaining apparent “sameness and difference of the same object in the same respect”? It explains the difference between two objects of, say, distinct shades of blue. Each has a property the other lacks. But how do we explain what two blue objects have in common, when they are not precisely the same shade of blue?

The answer will have to draw on the fact that though these two objects have the distinct properties, these properties, in turn, have something in common. They are both, say, shades of blue. The objects have in common the property of “having a shade of blue”. That is, what they share is “having one of the properties which is a shade of blue”.

Similarly, objects of quite different colours, say one grey and the other orange, will share “having a property which is a colour”. Being coloured is a property of individuals, and it amounts to the property of having a colour.

Similar observations may be made about pains, and other mental states. On one construal, functionalist theories of the mind identify the property of “having a pain” with the property of “having whatever property constitutes pain for that individual at that time”. This construes “having a pain” as a second-order property. Even if the property which
QUANTITIES

constitutes pain in dolphins is not the same property as the one which constitutes pain in people, nevertheless the dolphin-property and the people-property may have something in common, called their "functional role". And then, derivatively, pain-sufferers in general, whether dolphins or people, do have something in common. They have in common the second-order property of having a property with that "functional role".4

On this theory, then, there are three sets of properties to keep track of. There are the basic, first-order properties of individuals, on which all the others supervene. Then there are the properties of the first-order properties — these are second-degree properties. And finally, there are the second-order properties of individuals, the properties of "having some first-order properties which have such-and-such second-degree properties".

The basic puzzles about quantities are then explained in the following way. Objects with different "determinates" are different because each has a first-order property the other lacks; they are the same because they share the same second-order property.

This theory does not conflict with the original theory of determinables and determinates. It merely says more, in places where the original theory was silent. In particular, it says more about what a determinable is (viz. a second-order property), and how it relates to its determinates.

The original theory of determinates and determinables needs neither affirm nor deny that determinables are second-order properties. Yet there is good reason why the theory should be supplemented with this construal of determinables.

As Johnson pointed out, determinables and determinates stand in a very tight and characteristic pattern of logical relationships. If an object has a determinate, this entails that it has the corresponding determinable. But the reverse does not hold: possession of that determinable does not entail possession of that determinate. And yet, possession of that determinable does entail possession of one of the determinates falling under its scope. For instance, possession of mass does entail possession of either this specific mass, or that specific mass, or one of the other possible specific masses. A thing cannot just have mass, without having a particular mass.

This pattern of entailments cries out for explanation, and it was the
inability of the original theory of determinates and determinables to provide an explanation that lead us to point to its incompleteness. The second-order theory of determinables does furnish just such an explanation in a simple straight forward fashion. If you believe in determinables and determinates, and you reject the second-order story, then you need to supply an alternative explanation of the entailment pattern. No such alternative account of determinables is on the market. And besides, the explanation provided by the second-order theory just sounds terribly plausible. So we will take it that the theory of determinables and determinates can be construed in the second-order fashion sketched above.

We believe that this augmented theory of determinables and determinates is on the right track: but it is still incomplete. It is less incomplete than the platonic theory, and the original theory of determinates and determinables, but incomplete nevertheless. And in fact, the issue which the augmented theory of determinables and determinates overlooks, is one which platonic theories place on centre stage. This issue, is that of explaining how quantities can come in degrees. It is not the case, simply, that two objects are the same (both being, for instance, blue) and yet different (being different shades of blue): it is also the case that we can specify how different they are. We can say that two blue things are more different than two other blue things (with respect to colour). Similarly, two pains may be more alike than two other pains. And similarly also, a mass may be closer to one mass than to some other one. It is this feature which is so well exposed by the platonic degrees of participation or resemblance.

These facts about quantities are not easy to cope with, using even the augmented model of determinables and determinates. Some moves could be tried; but the point remains, that the augmented theory of determinables and determinates is incomplete. So we move on to explore another avenue.

THE RELATIONAL THEORY

Quantities generate relations among objects. At first sight, these relations among objects are just consequences of the intrinsic properties of the objects. For instance, consider the relation taller than. Goliath
stands in this relation to David. It is plausible to suggest that Goliath has an intrinsic property, his height, which he would have had whether David existed or not. Similarly David has an intrinsic property, quite independently of Goliath. Given the height of Goliath, and the height of David, it follows automatically that Goliath is taller than David.

There is no need to posit any intrinsic relation “taller than”, over and above the intrinsic properties of height. The relation supervenes on the properties.

Yet there is something unsatisfactory about this treatment of relations like “taller than”. There is, to begin with, a worry about whether there really is any such thing as an intrinsic property of height. It is tempting to suppose that the property rests on relations, rather than the other way around. Plausibly, being the height you are is nothing more than just being taller than certain things, and shorter than others.

However, we will not pursue this line of attack. It may in the end be possible to defend intrinsic properties of height, against the relationist critique. Nevertheless, even if intrinsic properties of height do exist, there remains a question about how such properties can ground relations between objects.

Consider the analysis of spatial relations, like “East of” for instance. Suppose we try to ground this relation in intrinsic properties of location. A is East of B, because A is here and B is there. But why should A’s being here, and B there, entail A’s being East of B? Only because here is a position which is East of the position which constitutes there. Using the positions of A and B, that ground their spatial relations, we presuppose the existence of spatial relations between their positions. Another famous case of this sort is furnished by time. The relation of earlier to later is surely not solely a product of the intrinsic characters of the related events. Relations are inescapable. They cannot be grounded in properties alone.

The same applies to a relation like “twice as massive as”. You may try to ground this in intrinsic properties, of determinate masses. But why should object A’s having one property, and object B’s having another property, entail A’s being twice as massive as B? We must presuppose a relation between the property of A and the property of B. The property of “having this mass” must stand in a relation of proportion to the property of “having that mass”. Otherwise there would be no
explanation of why objects with the first property are *more* massive than those with the second. And, indeed, there would be no explanation of why they are *that* much more massive than those with the second property.

Let us, then, abandon the attempt to ground quantities in properties alone. Let us start again.

In the platonic theory, individuals stand in a variety of relationships to a single thing (a Form, or Paradigm). Let us, instead, present them as standing in a variety of relationships to *one another*. This one is “twice as massive as” that one; that one is “one third as massive as” another one; and so on.

On this account, for an individual to have a particular determinate property is just for it to stand in a particular range of relationships to other individuals. For instance, to have *such-and-such* a mass is to stand in a certain set of relationships to other massive objects. For an object to be a particular shade of blue, is to stand in a set of relationships to other blue objects. And so on.

Such a battery of relations will make it easy to characterize one of the important features of quantities. When individuals possess a given quantity to distinct degrees, then there is an important way in which they *differ*. This is captured in the relational theory. Two such individuals differ, because each possesses different relationships to the other. For instance, we may have that *A* is taller than *B*, but of course *B* is not taller than *B*. So *A* and *B* differ because each has a (relational) property the other lacks.

This theory not only explains the presence of a difference between two objects: it also characterizes the different *kinds* of difference there can be. It can explain *how* different they can be. We may have, for instance *A* being twice as bright as *B*, and *C* three times as bright as *D*. Then, clearly, it is not enough to say just that *A* differs from *B*, and *C* differs from *D*. It is important to say that they differ differently. *C* differs more from *D* than *A* does from *B*. By appealing to two relationships, one between *A* and *B*, and the other between *C* and *D*, it is easy to capture the fact that the two pairs differ differently.

(In fact, to fully capture the way that *C* differs *more* from *D* than *A* does from *B*, we need to add an ingredient. We must add that the two relations, “twice as bright” and “three times as bright”, themselves stand
in a certain ordering relation. This same relationship holds between “three times as bright” and “six times as bright”. But a completely different relationship holds between the two relations “twice as bright” and “one third as bright”. Let us leave this complication for the moment.)

It is at this point that our theory again follows a path which is closely analogous to the orthodox treatment of mass-terms. An adequate theory of mass-terms must allow for the analysis of statements like “This glass has more water in it than that glass”. This requires us to deal with not only a class of individual samples of substances such as water, and not only with a common property of such samples, but also with important relations among these samples. These relations need to involve more than just similarity to a paradigm; they also need to impose an ordering on samples of the substance. Some samples include more, and others less, of the same substance.

The relational theory handles differences rather well; but does it adequately explain what individuals have in common? Things which have, for instance, different determinate masses do, nevertheless, have something in common.

We could simply add a property into the theory, a property which is shared by all the things which stand in mass-relations to one another. Call this property simply mass. When things stand in mass-relations to one another, this entails that they share the property of mass. And when two things share the property of mass, this entails that there is some mass-relation which holds between them.

This double-barrelled theory, containing many relations and a common property, is structurally similar to the Johnsonian theory of determinables and determinates. And, like that theory, it leaves a lot unexplained. It lays out a complex pattern of entailments, between the one common property and the many relations. Yet nothing has been said yet to explain why these entailments should hold.

These unexplained elements fall into place, if we supplement the relational theory with a specific account of the nature of the supposed common property. We need an account of the common property which displays a clear connection between it, and the many relations which underpin it.

The simplest way to deliver the required connections, is by constru-
ing the common property in a second-order manner. Let us explain, in
two stages. In the first stage, we will examine what objects with
different mass have in common; and in the second stage we will face the
question of how some can have more in common than others.

First, let us consider the many relations which are associated with a
quantity like mass. These will be relations like "more massive than",
"half as massive as", and so forth. It is plausible to postulate that each
of these relations share a common property. Each of them are relations
of mass.

So let us explore this idea further. The picture we are offered begins
with a profusion of different mass-relations among individuals. To this
we add a single property which all these relations share in common.
How does this property of relations explain what all massy objects have
in common? By the fact that its existence entails that all massy objects
share a certain second-order property. For any massy object, there are
some mass-relations it has to other things. (Indeed, there are mass-
relations it has to itself — "same mass as", for instance.) What all massy
objects have in common, is the property of standing in mass-relations
to something.

We may thus construe the common property of massiness, as a
second-order property — that of having mass-relations to things. This
construal has a large advantage over theories which give the common
property no analysis. It explains the pattern of entailments between the
common property of massiness, and the various mass-relations.

THE THREE-LEVEL THEORY

The relational theory is, nonetheless, still incomplete. What has not
yet been explained, is the way that some mass-relations are in some
sense "closer" to one another than to others.

Mass-relations all have something in common; and thereby, massy
objects have a second-order property in common. This is the first stage
in constructing a theory of quantities.

But there are complexities which are not fully captured by this first
stage. It enables us to explain how two things can differ (having different
masses) but yet also be the same (both having mass). However, it does
not explain how much two things may differ. It does not explain how a
thing may differ more from another, in mass, than it does from a third
thing. Nor does it explain how two things may differ in mass to just the same degree as they differ in, say, volume: one may be for instance be twice the mass and twice the volume of the other.

To obtain an adequate theory of quantities, we must thus introduce a second stage of construction. It is not sufficient to note that mass-relations all have something in common — or common property. We must also note that mass-relations can be more and less similar to one another. The relations “five times as long” and “six times as long” are in some sense closer to one another than either is to say “100 times as long”. Consider three pairs:

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\langle A, B \rangle \\
\langle C, D \rangle \\
\langle E, F \rangle
\]

suppose A is five times as long as B and C is six times as long as D and E is 100 times as long as F. Then there is an important respect in which the first two pairs are more similar to one another, than either is to the third.

Hence, we need to recognize relations among relations — relations of proportion.

The theoretical structure we offer, which in part is inspired by Frege\(^5\), thus rests on three fundamental ingredients: (1) individuals; (2) determinate relationships between individuals; (3) relations of proportion between those determinate relationships.

Thus, for instance, consider three individuals A, B and C, as our population at level (1). There will be a class of relations at level (2) holding among A, B and C. These level (2) relations may then be grouped, according to whether or not they stand in level (3) proportions to one another. All the “mass-relations” stand in proportions to one another; all the “volume-relations” stand in proportions to one another; but the mass-relations do not stand in proportions to the volume-relations, or vice versa.

Thus level (3) relations of proportion classify level (2) determinates into equivalence classes. Within each such equivalence class of level (2), the level (3) proportions will also impose an ordering. And it is this ordering which explains how one thing can be closer to a second in, say, mass, than it is to a third.

Furthermore, level (3) proportions can introduce finer distinctions
within equivalence classes at level (2). Level (3) proportions are universals: so they can be multiply instantiated. Thus, one and the same relation of proportion may hold between several distinct pairs of level (2) relations. For instance, several distinct pairs of mass-relations may stand in the same level (3) proportion.

Indeed, using level (3) proportions, we can generate classifications which cut across the equivalence classes — that is, which cut across the classes of mass-relations, volume-relations, velocity-relations, and so forth. For instance, two level (2) mass-relations will stand in some specific proportion; and two level (2) volume-relations may stand in exactly the same proportion.

This explains how it can be that when an object is “twice the mass of”, and also “twice the volume of” another, these two relations are at the same time both the same, and yet also different. These relations are different, in that one is a mass-relation, standing in proportions to other mass-relations, while the other is not a mass-relation, and stands in no proportions to mass-relations. And yet there is also something these two different relations have in common, as is reflected in the way we describe them: “twice the mass of” — ”twice the volume of”. This common factor, too, is explained by level (3) proportions. There is a proportion between “twice the mass of” and “the same mass as”; and this very same proportion holds also between “twice the volume of” and “the same volume as”. And that is why the same word “twice” occurs in the descriptions of both.

QUANTITY, NUMBER AND STRUCTURE

The cross-category similarities between determinate quantities helps to explain the usefulness of numbers in dealing with quantities. Given relations of proportion holding among mass-relations, for instance, we may generate a complex range of derivative properties and relations. For instance, an object may have a certain mass-relation to some designated object which determines our unit of measurement. Perhaps it has the relation “same mass as” to the unit. Thereby, it has a second-order property: the property of having the “same mass as” relation to the unit. When this is so, the object has a mass of 1 kg.

Suppose some other object has some different mass-relation to the
unit: it is as we say "twice as massive as" the unit. Thereby it has the property of having a mass of 2 kg. The number 2 appears here as a signal that the mass-relation between the object and an object of unit mass stands in the proportion of 2 to 1 to the "same mass as" relation; and the term "kg" specifies to which unit it stands in that proportion.

Earlier when we discussed the platonic (or berkeleyan) theory our objection was not that such a theory is mistaken, but only that it is incomplete. Objects do have relations to paradigms; and our theory not only allows, but also explains, how these relations arise. It is by way of such relations to paradigms that scales of measurement arise. Objects which specify standard units of a scale of measurement play the exact role of a paradigm in a platonic type theory.7

It may be worth noting that the level (3) relations among determinates may be more complex and discriminating for some classes of determinates than for others. Determinate physical quantities like mass stand in such a rich pattern of proportions to one another, that it forces us to draw upon the full resources of the real number system. In our terminology, however, pains and colours, too, count as quantities. It is highly likely that the level (3) relations among these quantities will manifest a variety of structures which are less linear, and less discriminating, and so which manifest structures other than that of the real number system.

Thus, quantities will subdivide into categories, according to the nature of the level (3) interrelations they manifest. These subdivisions correspond to the distinctions, familiar in measure theory, which are drawn between, for instance, interval and ratio scales of measurement.

Different sets of relations can manifest different patterns of proportions among their members. And different scales of measurement must have mathematical structures which reflect these different patterns. In cases like that of mass, the structure is in one sense very rich and discriminating; but it is also very regimented and linear. In cases such as pain, the proportions will be in certain respects much more complex. They may allow for non-linear partial orderings; incomparabilities across the orderings of different kinds of pains; and so forth. Dolphin pains may stand in the same sort of intensity-orderings, that human pains stand in; but it may not be possible to rank the intensity of a
dolphin pain against a human pain. Similarly, it may not be possible to rank some aches against some burning pains. Or even one person’s pains against another’s.

CONCLUSION

We have argued that, given the three levels — individuals; determinates; and proportions — we can account for the complex patterns of sameness and difference, which characterize quantities. The level (3) relations generate a rich network of second-order properties and relations among objects. And not only does it generate all these second-order properties and relations, but it also explains the pattern of entailment relations which hold among them.

We emphasise that the theory is not one without a justificatory path. The augmented theory of determinates and determinables (on which determinables are second order properties) explains the logical connection between the determinate properties and the determinable property, but fails to explain the relationships which the determinate properties have to each other. Moving to the relational theory and postulating that for an object to have a particular determinate property is in fact for this object to stand in a particular set of relationships to other objects, we preserve the second order account of determinable properties, but now explain the ordering relations that seem to hold between objects with related determinate properties. And finally in moving to the three-level theory where the relevant relations between the objects are themselves seen as being related, or standing in proportion to each other, we can explain not only the ordering of objects with the determinates, but give a full account of the logical relationships that hold between the determinate properties.

But not only do we hope that this justificatory path is persuasive, we hold out the challenge to find an alternative theory with as much explanatory power.

Nominalists will, of course, seek ways of avoiding the ontological cost of accepting not only relations, but also relations between relations. We do not pretend to have proved that realism is the only tenable position. But we do claim that, if you are to be a realist at all, then our three-tiered realism is to be recommended, because it does justice to
QUANTITIES

quantities to a greater extent than other realist alternatives. Furthermore, the three-tiered structure flows naturally from the spirit of realism. Realism takes its source from the recognition that things may be the same, and yet different. If such recognition justifies universals at all, then close attention to the complexities of such samenesses and differences leads straight to our three-tiered realism and to our three-level theory of quantities.

And finally, even if nominalism remains tempting to some, we claim that our realist theory has unifying and explanatory virtues which are very unlikely to be matched by nominalism. Almost inevitably, nominalists will be compelled to take a panoply of relational predicates as primitive. And worse, there will be complex patterns of entailments among these predicates, which will also have to be taken as primitive. In three-tiered realism, such predicates will be construed as second-order predicates, containing implicit quantification over both properties and proportions. And this construal will enable us to explain the entailments among them in a simple and natural manner. The three-level theory gives a much more explanatory account of quantities.

NOTES

5 G. Frege developed a theory of real numbers, which construes them as relations between relations, in Grundgesetze der Arithmetik, Volume II, Hermann Pohle, Jena, 1903. These relations between relations are construed as proportions holding between magnitudes (i.e. quantities, in our sense); ‘... exactly the same proportion of magnitudes which we find among line segments we should also have among durations, masses, light intensities, etc.’ (p. 156).
6 Frege's three-level theory, of relational quantities and proportions holding among them, was intended as a basis for a theory of real numbers. Such a theory of real numbers received formal elaboration by Whitehead and Russell, and by Quine; for an introductory exposition and further references see section 10 of W. V. Quine: 1966,

7 As Newton said, “By Number we understand not so much a multitude of Unities, as the abstracted Ratio of any Quantity, to another Quantity of the same kind, which we take for Unity.” (1728, Universal Arithmetik, 2nd edition, (tr. Ralphson, Longman, London), p. 2).

This view became orthodoxy; thus, Leonard Euler echoes Newton: “Mathematics, in general, is the science of quantity . . . Now we cannot measure or determine any quantity, except by considering some other quantity of the same kind as known, and pointing out their mutual relation . . . a number is nothing but the proportion of one magnitude to another arbitrarily assumed as the unit.” (1822, Elements of Algebra, 3rd edition, (tr. J. Hewlett, J. Johnson, London)).

The recent set-theoretical transformation of mathematics has obscured the relationship between number and physical quantities, and hence also the nature of quantities themselves. Our theory may be regarded as retrieving and up-dating lost wisdom, buried under many years of positivism and nominalism.

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